AMSCO'S

ALGEBRA 2 and TRIGONOMETRY

Ann Xavier Gantert

AMSCO SCHOOL PUBLICATIONS, INC.
315 HUDSON STREET, NEW YORK, N.Y. 10013

Dedication

To Jessica Alexander and Uriel Avalos in gratitude for their invaluable work in preparing this text for publication.

Ann Xavier Gantert

The author has been associated with mathematics education in New York State as a teacher and an author throughout the many changes of the past fifty years. She has worked as a consultant to the Mathematics Bureau of the Department of Education in the development and writing of Sequential Mathematics and has been a coauthor of Amsco's *Integrated Mathematics* series, which accompanied that course of study.

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Text Designer: Nesbitt Graphics, Inc.
Compositor: ICC Macmillan
Cover Design by Meghan J. Shupe
Cover Art by Radius Images (RM)

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When ordering this book, please specify:
R 159 P *or* ALGEBRA 2 AND TRIGONOMETRY, *Paperback* or
R 159 H *or* ALGEBRA 2 AND TRIGONOMETRY, *Hardbound*

ISBN 978-1-56765-703-6 (Paperback edition)
NYC Item 56765-703-5 (Paperback edition)

ISBN 978-1-56765-702-9 (Hardbound edition)
NYC Item 56765-702-8 (Hardbound edition)

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Printed in the United States of America

1 2 3 4 5 6 7 8 9 10 14 13 12 11 10 09 08

PREFACE

Algebra 2 and Trigonometry is a new text for a course in intermediate algebra and trigonometry that continues the approach that has made Amsco a leader in presenting mathematics in a modern, integrated manner. Over the last decade, this approach has undergone numerous changes and refinements to keep pace with ever-changing technology.

This textbook is the final book in the three-part series in which Amsco parallels the integrated approach to the teaching of high school mathematics promoted by the National Council of Teachers of Mathematics in its *Principles and Standards for School Mathematics* and mandated by the New York State Board of Regents in the *Mathematics Core Curriculum*. The text presents a range of materials and explanations that are guidelines for achieving a high level of excellence in their understanding of mathematics.

In this book:

- ✔ **The real numbers** are reviewed and the understanding of operations with irrational numbers, particularly radicals, is expanded.
- ✔ **The graphing calculator** continues to be used as a routine tool in the study of mathematics. Its use enables the student to solve problems that require computation that more realistically reflects the real world. The use of the calculator replaces the need for tables in the study of trigonometry and logarithms.
- ✔ **Coordinate geometry** continues to be an integral part of the visualization of algebraic and trigonometric relationships.
- ✔ **Functions** represent a unifying concept throughout. The algebraic functions introduced in *Integrated Algebra 1* are reviewed, and exponential, logarithmic, and trigonometric functions are presented.
- ✔ **Algebraic skills** from *Integrated Algebra 1* are maintained, strengthened, and expanded as both a holistic approach to mathematics and as a bridge to advanced studies.
- ✔ **Statistics** includes the use of the graphing calculator to reexamine range, quartiles, and interquartile range, to introduce measures of dispersion such as variance and standard deviation, and to determine the curve that best represents a set of bivariate data.

- ✔ **Integration** of geometry, algebra, trigonometry, statistics, and other branches of mathematics begun in *Integrated Algebra 1* and *Geometry* is continued and further expanded.

- ✔ **Exercises** are divided into three categories. *Writing About Mathematics* encourages the student to reflect on and justify mathematical conjectures, to discover counterexamples, and to express mathematical ideas in his or her own words. *Developing Skills* provides routine practice exercises that enable the student and teacher to evaluate the student's ability to both manipulate mathematical symbols and understand mathematical relationships. *Applying Skills* provides exercises in which the new ideas of each section, together with previously learned skills, are used to solve problems that reflect real-life situations.

- ✔ **Problem solving**, a primary goal of all learning standards, is emphasized throughout the text. Students are challenged to apply what has been learned to the solution of both routine and non-routine problems.

- ✔ **Enrichment** is stressed both in the text and in the Teacher's Manual where many suggestion are given for teaching strategies and alternative assessment. The Manual provides opportunities for extended tasks and hands-on activities. Reproducible *Enrichment Activities* that challenge students to explore topics in greater depth are provided in each chapter of the Manual.

In this text, the real number system is expanded to include the complex numbers, and algebraic, exponential, logarithmic, and trigonometric functions are investigated. The student is helped to understand the many branches of mathematics, to appreciate the common threads that link these branches, and to recognize their interdependence.

The intent of the author is to make this book of greatest service to the average student through detailed explanations and multiple examples. Each section provides careful step-by-step procedures for solving routine exercises as well as the non-routine applications of the material. Sufficient enrichment material is included to challenge students of all abilities.

Specifically:

- ✔ Concepts are carefully developed using appropriate language and mathematical symbolism. General principles are stated clearly and concisely.

- ✔ Numerous examples serve as models for students with detailed explanations of the mathematical concepts that underlie the solution. Alternative approaches are suggested where appropriate.

- ✔ Varied and carefully graded exercises are given in abundance to develop skills and to encourage the application of those skills. Additional enrichment materials challenge the most capable students.

This text is offered so that teachers may effectively continue to help students to comprehend, master, and enjoy mathematics as they progress in their education.

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